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SGP- LOCALLY CLOSED SETS IN TOPOLOGICAL SPACES (PART -II)

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Abstract

The aim of this paper is to introduce the new class of sgp- locally closed sets in topological spaces and studied some of their properties and characterizations.

Keywords *– Topological spaces, SGPLC(X,τ), SGPLC*(X,τ) SGPLC**(X,τ) LC(X,τ)*
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1 Introduction

The notion of a locally closed set in a topological space was introduced by Kuratowski and Sierpinski [8]. According to Bourbaki [5], a subset A of a topological space X is called locally closed in X if it is the intersection of an open set in X and a closed set in X. Ganster and Reilly [6] used locally closed sets to define LC- Continuity and LC-irresoluteness. Balachandran, Sundaram and Maki [3] introduced the concept of generalized locally closed sets in topological spaces and investigated some of their properties. Recently Sheik John [15] introduced the three new class of sets denoted by ω-LC(X, τ), ω-LC*(X, τ) and ω-LC**(X, τ) and each of which contains LC(X, τ). Also various authors like Gnanambal [7] and Park and Park [14] have introduced α-locally closed and semi generalized locally closed sets respectively in topological spaces.

2 Preliminaries

Throughout the thesis (X, τ) and (Y, σ) denote topological spaces on which no separation axioms are assumed unless explicitly stated and they simply written as X and Y respectively. All sets are considered to be subsets to topological spaces. The complement of A is denoted by X – A. The closure and interior of a set A are denoted by Cl(A) and int(A) respectively.

The following definitions are useful in the sequel :

DEFINITION 1.1 : A subset A of a space X is said to be

(i) Semi open [9] if A $\subset$ Cl (Int (A)).

(ii) semi-closed set[4] if Int(cl(A)) $\subseteq$ A.

(iii) preopen [5] ifA $\subset$ Int (Cl (A))

(iv) preclosed [12] ifCl (Int (A))$\subseteq$ A

(v) α - open [13] if A $\subset$ Int (Cl (Int A)))

(vi) α - closed [11] ifCl (Int (Cl (A)))$\subseteq$ A

(vii) Semi - preopen [2] (= β - open [1]) if A $\subset$ Cl (Int (Cl (A)))

(viii) a semi- pre closed set [1] if Int(cl(Int(A))) $\subset$ A

The family of all semi open sess (resp. semi-pre open sets) of X will be denoted by SO(X) SPO(X).

1.2 sgp-Locally Closed Sets

In this section, we introduce sgp-locally closed sets and sgp-submaximal and study some of their properties.

Definition 1.2.1: A subset A of a topological space (X, τ) is called a semi-generalized-pre locally closed set (briefly sgplc-set) if A = S $\cap$ F where S is sgp-open and F is sgp-closed.

The class of all semi-generalized-pre locally closed sets in (X,τ) is denoted by SGPLC(X,τ).

Definition 1.2.2: A subset A of a topological space (X,τ) is said to be SGPLC*-set if there exist sgp-open set S and a closed set F of (X,τ) such that A = S $\cap$ F.

Definition 1.2.3: A subset A of a topological space (X,τ) is said to be SGPLC**-set if there exist an open set S and a sgp-closed set F of (X,τ) such that A = S$\cap$ F.

Theorem 1.2.4: For a subset A of (X,τ), the following are equivalent:

1) A ∈ SGPLC*(X, τ)
2) A = P ∩ pCl (A) for some sgp-open set P.
3) pCl (A)-A is sgp-closed.
4) A ∪ (X-pCl(A)) is sgp-open.

Proof: (1) ⇒ (2):- Let A∈ SGPLC* (X,τ). Then there exists a sgp-open set P and a closed set F of (X,τ) such that A = P∩ F. Since A ⊆ P and A ⊆ pCl(A) . Therefore we have A ⊆ P ∩ pCl(A).

Conversely, since pCl(A) ⊆ F , P ∩ pCl(A) ⊆ P ∩ F= A . Which implies that A = P ∩ pCl(A).

(2) ⇒ (1):- Since P is sgp-open and pCl(A) is closed.

P ∩ pCl(A) ∈ SGPLC* (X,τ).Which implies that A∈ SGPLC* (X,τ).

(3) ⇒ (4) :- Let F = pCl(A)-A. Then F is sgp-closed by the assumption and X – F = X ∩ (X – (pCl(A) – A)) = A ∪ (X-pCl(A)). But X-F is sgp-open. This shows that A ∪ (X - pCl(A)) is sgp-open .

(4) ⇒ (3):- Let U = A ∪ (X-pCl(A)). Since U is sgp-open, X-U is sgp-closed. X – U = X- (A ∪ (X – pCl(A))) = pCl(A) ∩ (X-A) =pC(A) – A.

Thus pCl(A) – A is sgp-closed set .

(4) ⇒ (2):- Let P = A ∪ (X – pCl(A)) Thus P is sgp-open . We prove that A = P ∩ pCl(A) for some sgp-open set P. P ∩ pCl(A) = (A ∪ (X-pCl(A))) ∩ pCl(A) = (pCl(A) ∩ A) ∪ (pCl(A) ∩ (X – pCl(A))) = A ∪ ϕ = A. Therefore A = P ∩ pCl (A).

(2) ⇒ (4):- Let A = P ∩ pCl (A) for some sgp-open set P. Then we prove that A ∪ (X-pCl(A)) is sgp-open. Now A ∪ (X-pCl(A)) = (P ∩ pCl(A)) ∪ (X – pCl(A)) = P ∩ (pCl(A) ∪ (X – pCl(A))) = P. Which is sgp-open. Thus A ∪ (X – pCl(A)) is sgp-open .

Theorem 1.2.5: If A, B∈ SGPLC (X,τ), then A ∩ B ∈ SGPLC (X,τ).

Proof: From the assumptions, there exist sgp-open sets P and Q such that A = P ∩ pCl (A) and B = Q ∩ pCl (B). Then A ∩ B = (P ∩ Q) ∩ (pCl (A) ∩ pCl(B)). Since P ∩ Q is sgp-open set and pCl (A) ∩ pCl (B) is closed. Therefore A ∩ B ∈ SGPLC (X,τ).

Theorem 1.2.6: If A ∈SGPLC (X,τ) and B is sgp-closed set in (X,τ), then A ∩ B ∈ SGPLC (X,τ).

Proof: Since A $\in$ SGPLC (X,τ), there exist a sgp-open set P and a sgp-closed set Q such that A = P $\cap$ Q. Now A $\cap$ B = (P $\cap$ Q) $\cap$ B = P $\cap$(Q $\cap$ B). Since P is sgp-open and Q $\cap$ B is sgp-closed, Therefore A $\cap$ B $\in$ SGPLC (X,τ).

Theorem 1.2.7: If A $\in$SGPLC*(X,τ) and B is sgp-open (or closed) set in (X,τ), then A $\cap$ B $\in$ SGPLC*(X,τ).

Proof: Since A $\in$ SGPLC*(X,τ), there exist a sgp-open set P and a closed set Q such that A = P $\cap$ Q. Now A $\cap$ B = (P $\cap$ Q) $\cap$ B = (P $\cap$ B) $\cap$ Q. Since P $\cap$ B is sgp-open and Q is closed, it follows that A $\cap$ B $\in$ SGPLC* (X,τ).

In this case of B being a closed set, we have A $\cap$ B = (P $\cap$ Q) $\cap$ B = P$\cap$ (Q $\cap$ B). Since P is sgp-open set and Q $\cap$ B is closed. Thus A $\cap$ B $\in$ SGPLC* (X,τ).

Theorem 1.2.8: If A $\in$SGPLC**(X,τ) and B is sgp-closed (resp. open) set in (X,τ), then A $\cap$ B $\in$ SGPLC**(X,τ).

Proof: Since A $\in$ SGPLC**(X,τ), there exist an open set P and a sgp-closed set Q such that A = P $\cap$ Q. Now A $\cap$ B = (P $\cap$ Q) $\cap$ B = P $\cap$(Q $\cap$ B). Since P is open and Q$\cap$B is sgp-closed, Therefore A $\cap$ B $\in$ SGPLC**(X,τ).

In this case of B being an open set, we have A $\cap$ B = (P $\cap$ Q) $\cap$B = (P $\cap$B) $\cap$Q. Since P $\cap$B is open and Q is sgp-closed, Thus A $\cap$ B $\in$ SGPLC**(X,τ).

Theorem 1.2.9: Let (X,τ) and (Y, σ) be topological spaces.

1) If A $\in$ SGPLC (X,τ) and B $\in$ SGPLC (Y, σ), then A $\times$ B $\in$ SGPLC $(X \times Y, \tau \times \sigma)$
2) If A $\in$ SGPLC*(X,τ) and B $\in$ SGPLC*(Y, σ), then A $\times$ B $\in$ SGPLC* $(X \times Y, \tau \times \sigma)$.
3) If A $\in$ SGPLC**(X,τ) and B $\in$ SGPLC** (Y, σ), then A $\times$ B $\in$ SGPLC**$(X \times Y, \tau \times \sigma)$.

Proof: 1) Let A $\in$ SGPLC (X,τ) and B $\in$ SGPLC (Y, σ). Then there exist sgp-open sets M and M^l of (X,τ) and (Y, σ) and sgp-closed sets N and N^l of X and Y respectively such that A= M $\cap$ N and B = $M^l \cap N^l$.

Then A $\times$ B = $(M \times M^l) \cap (N \times N^l)$ holds. Hence A $\times$ B $\in$ SGPLC $(X \times Y, \tau \times \sigma)$.

2) Let A $\in$ SGPLC*(X,τ) and B $\in$ SGPLC*(Y, σ). Then there exist sgp-open sets K and K^l of (X,τ) and (Y, σ) and sgp-closed sets L and L^l of X and Y respectively such that A= K $\cap$ L and B = $K^l \cap L^l$.

Then A $\times$ B = $(K \times K^l) \cap (L \times L^l)$ holds. Hence A $\times$ B $\in$ SGPLC*$(X \times Y, \tau \times \sigma)$.

3) Let A ∈ SGPLC** (X,τ) and B ∈ SGPLC**(Y, σ). Then there exist open sets W and W^l of (X,τ) and (Y, σ) and sgp-closed sets V and V^l of X and Y respectively such that A= W ∩ V and B = $W^l \cap V^l$.

Then $A \times B = (W \times W^l) \cap (V \times V^l)$ holds.

Hence A × B ∈ SGPLC**(X × Y, τ × σ).

Definition 1.2.10: A topological space (X,τ) is said to be sgp-submaximal if every dense subset in it is sgp-open.

Theorem 1.2.11: Every submaximal space is sgp-submaximal.

Proof: Let (X,τ) be a submaximal space and A be a dense subset of (X,τ). Then A is open. But every open set is sgp-open and so A is sgp-open. Therefore (X,τ) is sgp-submaximal.

The converse of the above theorem need not be true as seen from the following example.

Example 1.2.12: In the Example 6.2.11, the space (X,τ) is sgp-submaximal but not submaximal, every dense subset is sgp-open. However the set A= {a, b} is dense in (X,τ), but it is not open in X. Therefore (X,τ) is not submaximal.

Theorem 1.2.13: Every ω-submaximal space is sgp-submaximal.

Proof: Let (X,τ) be a ω-submaximal space and A be a dense subset of (X,τ). Then A is ω-open. But every ω-open set is sgp-open and so A is sgp-open. Therefore (X,τ) is sgp-submaximal.

The converse of the above theorem need not be true as seen from the following example.

Example 1.2.14: Let X = {a, b, c} and τ = {X, ϕ, {a}}. Then the space (X,τ) is sgp-submaximal but not an ω-submaximal.

Remark 1.2.15: g-submaximals and sgp-submaximals are independent as seen from the following examples.

Example 1.2.16: In the Example 6.2.31, the space (X,τ) is g-submaximal but not a sgp-submaximal, because for the subset {a, c} is dense in (X,τ) it is not a sgp-open set in (X,τ) but it is g-open in (X,τ).

Example 1.2.17: Let X = {a, b, c} and τ = {X, ϕ, {a, b}}. Then the space (X,τ) is sgp-submaximal but not a g-submaximal, because for the subset {b, c} is dense in (X,τ) it is not a g-open set in (X,τ) but it is sgp-open in (X,τ).

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